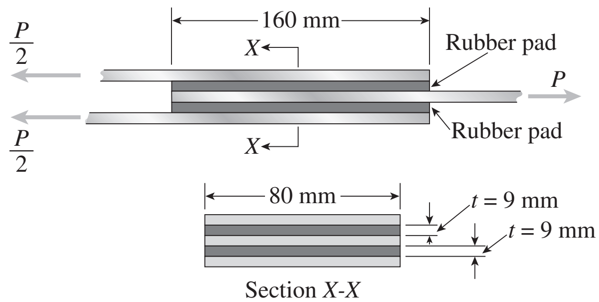
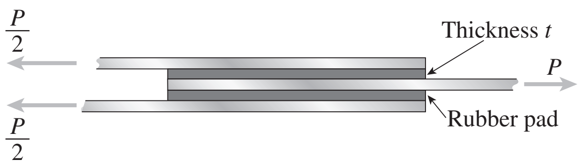
Problem 1.6-10 A flexible connection consisting of rubber pads (thickness $t = 9$ mm) bonded to steel plates is shown in the figure. The pads are 160 mm long and 80 mm wide.

(a) Find the average shear strain γ_{aver} in the rubber if the force $P = 16$ kN and the shear modulus for the rubber is $G = 1250$ kPa.
(b) Find the relative horizontal displacement δ between the interior plate and the outer plates.



Solution 1.6-10 **Rubber pads bonded to steel plates**



Rubber pads: $t = 9\,\text{mm}$

Length $L = 160\,\text{mm}$

Width $b = 80\,\text{mm}$

$G = 1250\,\text{kPa}$

$P = 16\,\text{kN}$

(a) Shear stress and strain in the rubber pads

$$\tau_{aver} = \frac{P/2}{bL} = \frac{8\text{ kN}}{(80\text{ mm})(160\text{ mm})} = 625\text{ kPa}$$

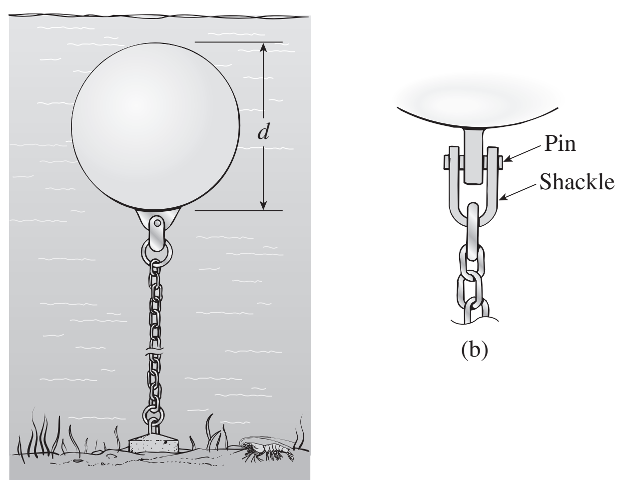
$$\gamma_{aver} = \frac{\tau_{aver}}{G} = \frac{625\text{ kPa}}{1250\text{ kPa}} = 0.50 \quad \longleftarrow$$

(b) Horizontal displacement

$$\delta = \gamma_{aver} t = (0.50)(9\,\text{mm}) = 4.50\,\text{mm} \quad \longleftarrow$$

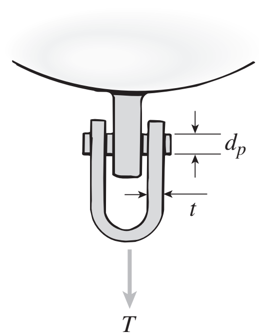
Problem 1.6-11 A spherical fiberglass buoy used in an underwater experiment is anchored in shallow water by a chain [see part (a) of the figure]. Because the buoy is positioned just below the surface of the water, it is not expected to collapse from the water pressure. The chain is attached to the buoy by a shackle and pin [see part (b) of the figure]. The diameter of the pin is 0.5 in. and the thickness of the shackle is 0.25 in. The buoy has a diameter of 60 in. and weighs 1800 lb on land (not including the weight of the chain).

(a) Determine the average shear stress τ_{aver} in the pin.
(b) Determine the average bearing stress σ_b between the pin and the shackle.



(a) (b)

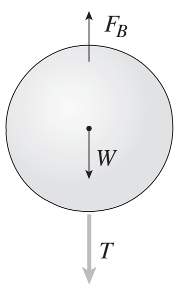
Solution 1.6-11 Submerged buoy



d = diameter of buoy
= 60 in.
T = tensile force in chain
d_p = diameter of pin
= 0.5 in.
t = thickness of shackle
= 0.25 in.
W = weight of buoy
= 1800 lb

γ_W = weight density of sea water
= 63.8 lb/ft³

FREE-BODY DIAGRAM OF BUOY



F_B = buoyant force of water pressure (equals the weight of the displaced sea water)

V = volume of buoy

$$= \frac{\pi d^3}{6} = 65.45 \text{ ft}^3$$

$$F_B = \gamma_W \quad V = 4176 \text{ lb}$$

EQUILIBRIUM

$$T = F_B - W = 2376 \text{ lb}$$

(a) AVERAGE SHEAR STRESS IN PIN

A_p = area of pin

$$A_p = \frac{\pi}{4} d_p^2 = 0.1963 \text{ in.}^2$$

$$\tau_{\text{aver}} = \frac{T}{2A_p} = 6050 \text{ psi} \quad \longleftarrow$$

(b) BEARING STRESS BETWEEN PIN AND SHACKLE

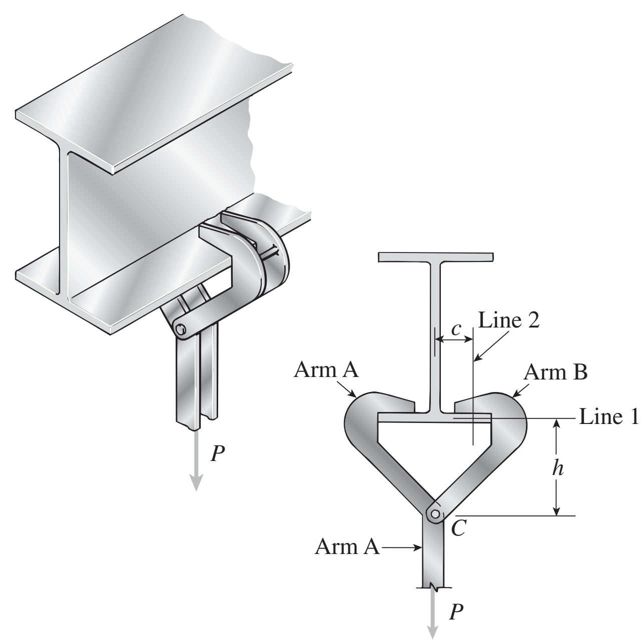
$$A_b = 2d_p t = 0.2500 \text{ in.}^2$$

$$\sigma_b = \frac{T}{A_b} = 9500 \text{ psi} \quad \longleftarrow$$

Problem 1.6-12 The clamp shown in the figure is used to support a load hanging from the lower flange of a steel beam. The clamp consists of two arms (A and B) joined by a pin at C. The pin has diameter d = 12 mm. Because arm B straddles arm A, the pin is in double shear.

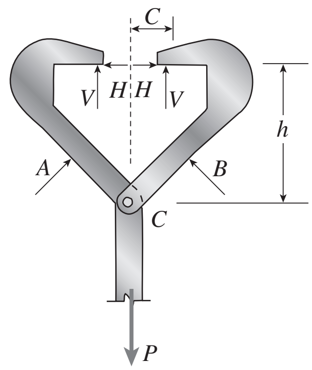
Line 1 in the figure defines the line of action of the resultant horizontal force H acting between the lower flange of the beam and arm B. The vertical distance from this line to the pin is h = 250 mm. Line 2 defines the line of action of the resultant vertical force V acting between the flange and arm B. The horizontal distance from this line to the centerline of the beam is c = 100 mm. The force conditions between arm A and the lower flange are symmetrical with those given for arm B.

Determine the average shear stress in the pin at C when the load P = 18 kN.



Solution 1.6-12 Clamp supporting a load P

FREE-BODY DIAGRAM OF CLAMP



$h = 250\,\text{mm}$

$c = 100\,\text{mm}$

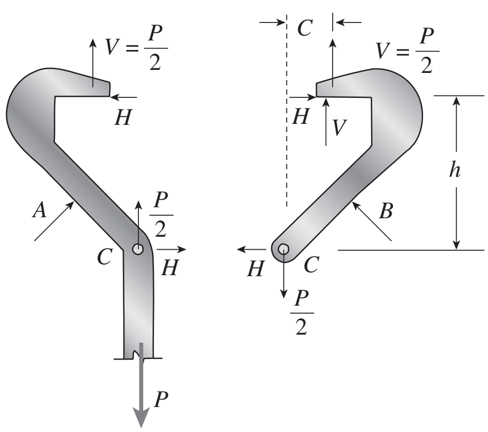
$P = 18\,\text{kN}$

From vertical equilibrium:

$$V = \frac{P}{2} = 9\text{ kN}$$

d = diameter of pin at C = 12 mm

FREE-BODY DIAGRAMS OF ARMS A AND B

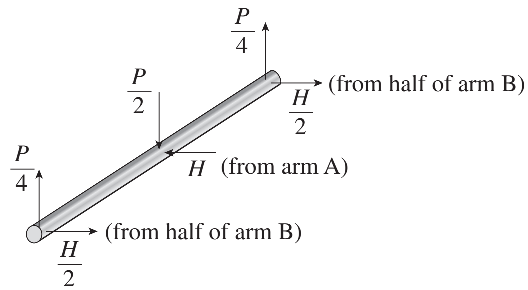


$$\Sigma M_c = 0$$

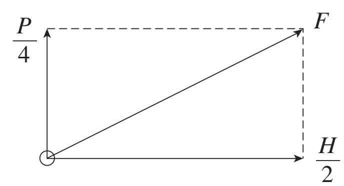
$$Vc - Hh = 0$$

$$H = \frac{V_c}{h} = \frac{P_c}{2h} = 3.6\text{ kN}$$

FREE-BODY DIAGRAM OF PIN



SHEAR FORCE F IN PIN



$$F = \sqrt{\left(\frac{P}{4}\right)^2 + \left(\frac{H}{2}\right)^2}$$

$$= 4.847\,\text{kN}$$

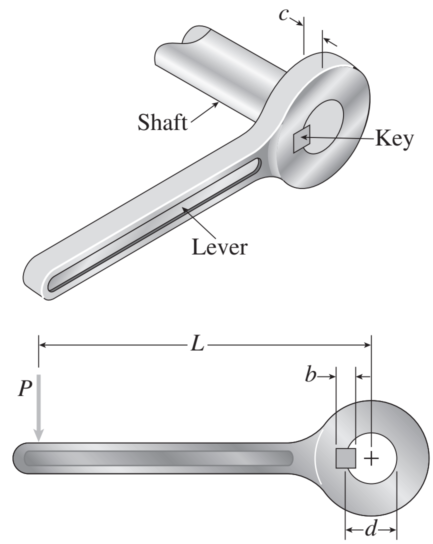
AVERAGE SHEAR STRESS

$$\tau_{\text{aver}} = \frac{F}{A_{\text{pin}}} = \frac{F}{\frac{\pi d^2}{4}} = 42.9\text{ MPa} \quad \longleftarrow$$

Problem 1.6-13 A specially designed wrench is used to twist a circular shaft by means of a square key that fits into slots (or *keyways*) in the shaft and wrench, as shown in the figure. The shaft has diameter d, the key has a square cross section of dimensions $b \times b$, and the length of the key is c. The key fits half into the wrench and half into the shaft (i.e., the keyways have a depth equal to $b/2$).

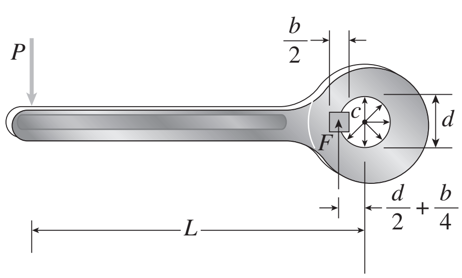
Derive a formula for the average shear stress τ_{aver} in the key when a load P is applied at distance L from the center of the shaft.

Hints: Disregard the effects of friction, assume that the bearing pressure between the key and the wrench is uniformly distributed, and be sure to draw free-body diagrams of the wrench and key.



Solution 1.6-13 Wrench with keyway

FREE-BODY DIAGRAM OF WRENCH



With friction disregarded, the bearing pressures between the wrench and the shaft are radial. Because the bearing pressure between the wrench and the key is uniformly distributed, the force F acts at the midpoint of the keyway.

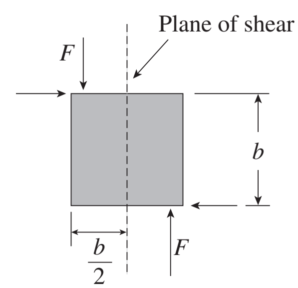
(width of keyway = $b/2$)

$\Sigma M_c = 0$ ↺+ ↻

$$PL - F\left(\frac{d}{2} + \frac{b}{4}\right) = 0$$

$$F = \frac{4PL}{2d + b}$$

FREE-BODY DIAGRAM OF KEY



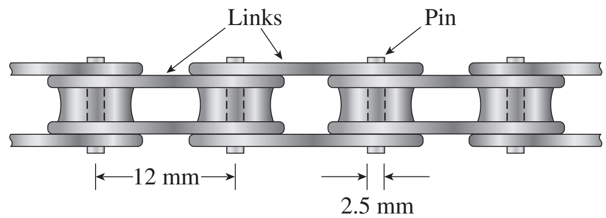
$$\tau_{aver} = \frac{F}{bc}$$

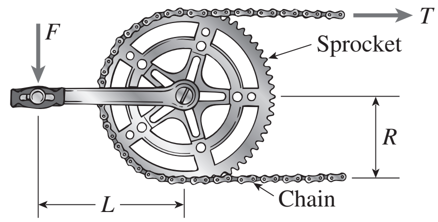
$$= \frac{4PL}{bc(2d + b)} \quad \longleftarrow$$

Problem 1.6-14 A bicycle chain consists of a series of small links, each 12 mm long between the centers of the pins (see figure). You might wish to examine a bicycle chain and observe its construction. Note particularly the pins, which we will assume to have a diameter of 2.5 mm.

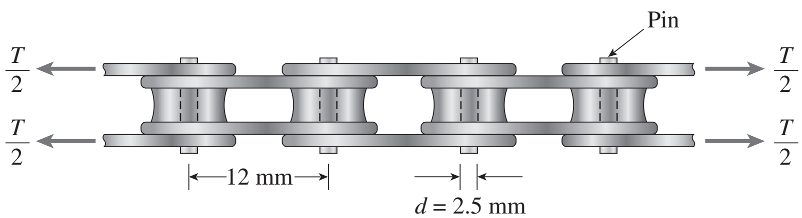
In order to solve this problem, you must now make two measurements on a bicycle (see figure): (1) the length L of the crank arm from main axle to pedal axle, and (2) the radius R of the sprocket (the toothed wheel, sometimes called the chainring).

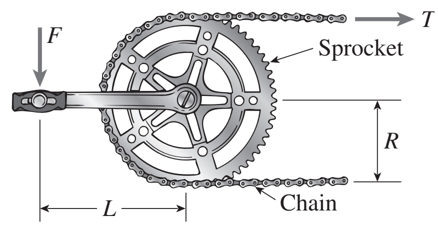
(a) Using your measured dimensions, calculate the tensile force T in the chain due to a force $F = 800$ N applied to one of the pedals.
(b) Calculate the average shear stress τ_{aver} in the pins.





Solution 1.6-14 **Bicycle chain**





F = force applied to pedal = 800 N

L = length of crank arm

R = radius of sprocket

MEASUREMENTS (FOR AUTHOR'S BICYCLE)

(1) $L = 162$ mm (2) $R = 90$ mm

(a) TENSILE FORCE T IN CHAIN

$$\Sigma M_{axle} = 0 \quad FL = TR \quad T = \frac{FL}{R}$$

Substitute numerical values:

$$T = \frac{(800\ N)(162\ \text{mm})}{90\ \text{mm}} = 1440\ \text{N} \quad \longleftarrow$$

(b) SHEAR STRESS IN PINS

$$\tau_{aver} = \frac{T/2}{A_{pin}} = \frac{T}{2\left(\frac{\pi d^2}{4}\right)} = \frac{2T}{\pi d^2}$$

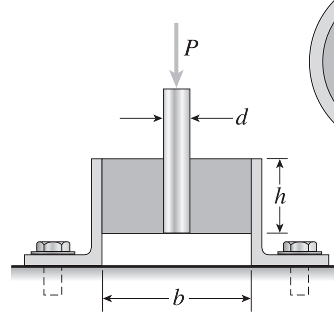
$$= \frac{2FL}{\pi d^2 R}$$

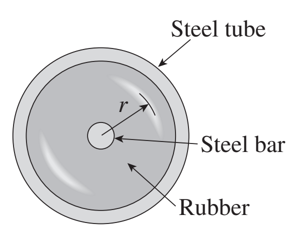
Substitute numerical values:

$$\tau_{aver} = \frac{2(800\ \text{N})(162\ \text{mm})}{\pi(2.5\ \text{mm})^2(90\ \text{mm})} = 147\ \text{MPa} \quad \longleftarrow$$

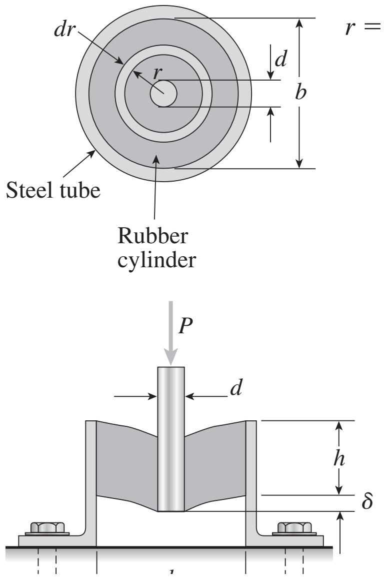
Problem 1.6-15 A shock mount constructed as shown in the figure is used to support a delicate instrument. The mount consists of an outer steel tube with inside diameter b, a central steel bar of diameter d that supports the load P, and a hollow rubber cylinder (height h) bonded to the tube and bar.

(a) Obtain a formula for the shear stress τ in the rubber at a radial distance r from the center of the shock mount.
(b) Obtain a formula for the downward displacement δ of the central bar due to the load P, assuming that G is the shear modulus of elasticity of the rubber and that the steel tube and bar are rigid.





Solution 1.6-15 Shock mount



r = radial distance from center of shock mount to element of thickness dr

(a) Shear stress τ at radial distance r

A_s = shear area at distance r

$$= 2\pi rh$$

$$\tau = \frac{P}{A_s} = \frac{P}{2\pi rh} \quad \longleftarrow$$

(b) Downward displacement δ

γ = shear strain at distance r

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi rhG}$$

$d\delta$ = downward displacement for element dr

$$d\delta = \gamma dr = \frac{Pdr}{2\pi rhG}$$

$$\delta = \int d\delta = \int_{d/2}^{b/2} \frac{Pdr}{2\pi rhG}$$

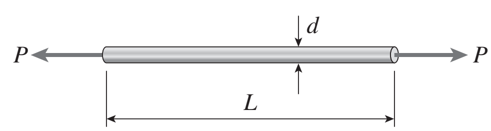
$$\delta = \frac{P}{2\pi hG}\int_{d/2}^{b/2}\frac{dr}{r} = \frac{P}{2\pi hG}[\ln r]_{d/2}^{b/2}$$

$$\delta = \frac{P}{2\pi hG}\ln\frac{b}{d} \quad \longleftarrow$$

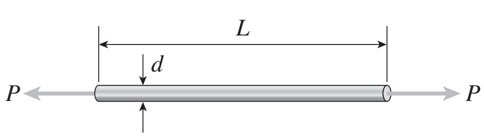
Allowable Loads

Problem 1.7-1 A bar of solid circular cross section is loaded in tension by forces P (see figure). The bar has length $L = 16.0$ in. and diameter $d = 0.50$ in. The material is a magnesium alloy having modulus of elasticity $E = 6.4 \times 10^6$ psi. The allowable stress in tension is $\sigma_{allow} = 17{,}000$ psi, and the elongation of the bar must not exceed 0.04 in.

What is the allowable value of the forces P?



Solution 1.7-1 Magnesium bar in tension



$L = 16.0$ in.

$d = 0.50$ in.

$E = 6.4 \times 10^6$ psi

$\sigma_{allow} = 17{,}000$ psi $\quad \delta_{max} = 0.04$ in.

Maximum load based upon elongation

$$t_{max} = \frac{\delta_{max}}{L} = \frac{0.04\text{ in.}}{16\text{ in.}} = 0.00250$$

$$\sigma_{max} = Et_{max} = (6.4 \times 10^6\text{ psi})(0.00250)$$

$$= 16{,}000\text{ psi}$$

$$P_{max} = \sigma_{max}\ A = (16{,}000\text{ psi})\left(\frac{\pi}{4}\right)(0.50\text{ in.})^2$$

$$= 3140\text{ lb}$$

Maximum load based upon tensile stress

$$P_{max} = \sigma_{allow}\ A = (17{,}000\text{ psi})\left(\frac{\pi}{4}\right)(0.50\text{ in.})^2$$

$$= 3340\text{ lb}$$

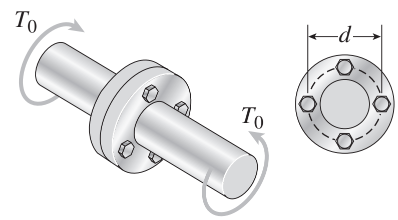
Allowable load

Elongation governs.

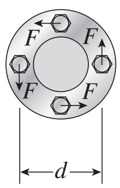
$$P_{allow} = 3140\text{ lb} \quad \longleftarrow$$

Problem 1.7-2 A torque T_0 is transmitted between two flanged shafts by means of four 20-mm bolts (see figure). The diameter of the bolt circle is $d = 150$ mm.

If the allowable shear stress in the bolts is 90 MPa, what is the maximum permissible torque? (Disregard friction between the flanges.)



Solution 1.7-2 Shafts with flanges



T_o = torque transmitted by bolts

d_B = bolt diameter = 20 mm

d = diameter of bolt circle

= 150 mm

$\tau_{allow} = 90$ MPa

F = Shear force in one bolt

$$T_o = 4F\left(\frac{d}{2}\right) = 2Fd$$

Allowable shear force in one bolt

$$F = \tau_{allow}A_{bolt} = (90\text{ MPa})\left(\frac{\pi}{4}\right)(20\text{ mm})^2$$

$$= 28.27\text{ kN}$$

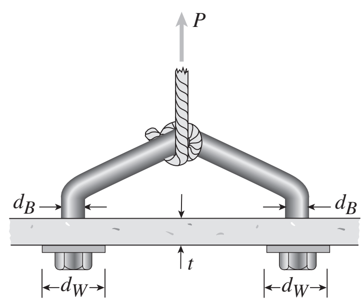
Maximum torque

$$T_o = 2Fd = 2(28.27\text{ kN})(150\text{ mm})$$

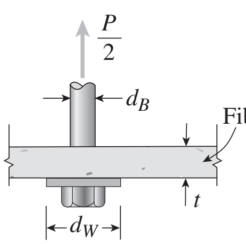
$$= 8.48\text{ kN}\cdot\text{m} \longleftarrow$$

Problem 1.7-3 A tie-down on the deck of a sailboat consists of a bent bar bolted at both ends, as shown in the figure. The diameter d_B of the bar is ¼ in., the diameter d_W of the washers is ⅞ in., and the thickness t of the fiberglass deck is ⅜ in.

If the allowable shear stress in the fiberglass is 300 psi, and the allowable bearing pressure between the washer and the fiberglass is 550 psi, what is the allowable load P_{allow} on the tie-down?



Solution 1.7-3 Bolts through fiberglass



$$d_s = \frac{1}{4}\text{ in.}$$

$$d_W = \frac{7}{8}\text{ in.}$$

$$t = \frac{3}{8}\text{ in.}$$

Allowable load based upon shear stress in fiberglass

$\tau_{allow} = 300$ psi

Shear area $A_s = \pi d_W t$

$$\frac{P_1}{2} = \tau_{allow}A_s = \tau_{allow}(\pi d_W t)$$

$$= (300\text{ psi})(\pi)\left(\frac{7}{8}\text{ in.}\right)\left(\frac{3}{8}\text{ in.}\right)$$

$$= 309.3\text{ lb}$$

$$P_1 = 619\text{ lb}$$

Allowable load based upon bearing pressure

$\sigma_b = 550$ psi

Bearing area $A_b = \frac{\pi}{4}(d_W^2 - d_B^2)$

$$\frac{P_2}{2} = \sigma_b A_b = (550\text{ psi})\left(\frac{\pi}{4}\right)\left[\left(\frac{7}{8}\text{ in.}\right)^2 - \left(\frac{1}{4}\text{ in.}\right)^2\right]$$

$$= 303.7\text{ lb}$$

$$P_2 = 607\text{ lb}$$

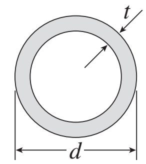
Allowable load

Bearing pressure governs.

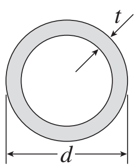
$$P_{allow} = 607\text{ lb} \longleftarrow$$

Problem 1.7-4 An aluminum tube serving as a compression brace in the fuselage of a small airplane has the cross section shown in the figure. The outer diameter of the tube is $d = 25$ mm and the wall thickness is $t = 2.5$ mm. The yield stress for the aluminum is $\sigma_Y = 270$ MPa and the ultimate stress is $\sigma_U = 310$ MPa.

Calculate the allowable compressive force P_{allow} if the factors of safety with respect to the yield stress and the ultimate stress are 4 and 5, respectively.



Solution 1.7-4 Aluminum tube in compression



$d = 25$ mm

$t = 2.5$ mm

d_o = inner diameter

$= 20$ mm

$$A_{tube} = \frac{\pi}{4}(d^2 - d_o^2) = 176.7 \text{ mm}^2$$

YIELD STRESS

$\sigma_Y = 270$ MPa

F.S. = 4

$$\sigma_{allow} = \frac{270 \text{ MPa}}{4} = 67.5 \text{ MPa}$$

ULTIMATE STRESS

$\sigma_U = 310$ MPa

F.S. = 5

$$\sigma_{allow} = \frac{310 \text{ Mpa}}{5} = 62 \text{ MPa}$$

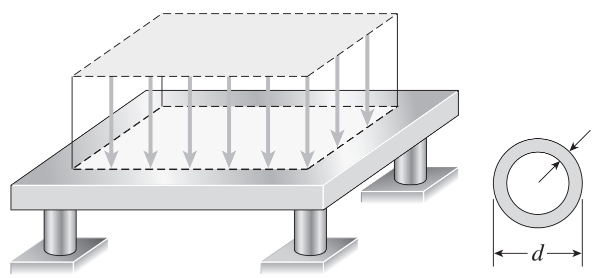
The ultimate stress governs.

ALLOWABLE COMPRESSIVE FORCE

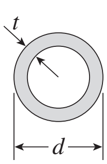
$$P_{allow} = \sigma_{allow} A_{tube} = (62 \text{ MPa})(176.7 \text{ mm}^2) = 11.0 \text{ kN} \longleftarrow$$

Problem 1.7-5 A steel pad supporting heavy machinery rests on four short, hollow, cast iron piers (see figure). The ultimate strength of the cast iron in compression is 50 ksi. The outer diameter of the piers is $d = 4.5$ in. and the wall thickness is $t = 0.40$ in.

Using a factor of safety of 3.5 with respect to the ultimate strength, determine the total load P that may be supported by the pad.



Solution 1.7-5 Cast iron piers in compression



Four piers

$\sigma_U = 50$ ksi

$n = 3.5$

$$\sigma_{allow} = \frac{\sigma_U}{n} = \frac{50 \text{ ksi}}{3.5} = 14.29 \text{ ksi}$$

$d = 4.5$ in.

$t = 0.4$ in.

$d_o = 3.7$ in.

$$A = \frac{\pi}{4}(d^2 - d_o^2) = \frac{\pi}{4}[(4.5 \text{ in.})^2 - (3.7 \text{ in.})^2]$$

$= 5.152$ in.^{2}

P_1 = allowable load on one pier

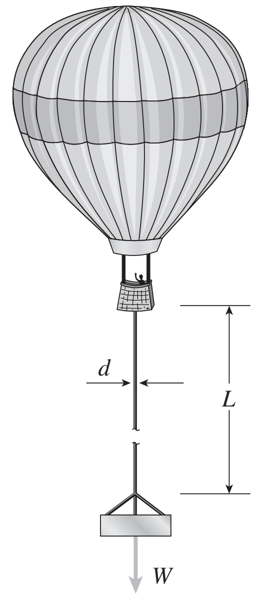
$= \sigma_{allow} A = (14.29 \text{ ksi})(5.152 \text{ in.}^2)$

$= 73.62$ k

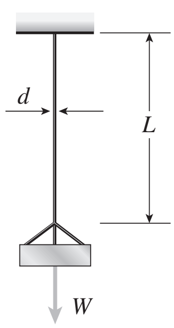
Total load $P = 4 \quad P_1 = 294$ k

Problem 1.7-6 A long steel wire hanging from a balloon carries a weight W at its lower end (see figure). The 4-mm diameter wire is 25 m long.

What is the maximum weight W_{max} that can safely be carried if the tensile yield stress for the wire is $\sigma_Y = 350$ MPa and a margin of safety against yielding of 1.5 is desired? (Include the weight of the wire in the calculations.)



Solution 1.7-6 **Wire hanging from a balloon**



$d = 4.0$ mm

$L = 25$ m

$\sigma_Y = 350$ MPa

Margin of safety = 1.5

Factor of safety = $n = 2.5$

$$\sigma_{allow} = \frac{\sigma_Y}{n} = 140 \text{ MPa}$$

Weight density of steel: $\gamma = 77.0$ kN/m^3

Weight of wire:

$$W_o = \gamma A L = \gamma\left(\frac{\pi d^2}{4}\right)(L)$$

$$W_o = (77.0 \text{ kN/m}^3)\left(\frac{\pi}{4}\right)(4.0 \text{ mm})^2(25 \text{ m})$$

$$= 24.19 \text{ N}$$

Total load $P = W_{max} + W_o = \sigma_{allow} A$

$$W_{max} = \sigma_{allow} A - W_o$$

$$= (140 \text{ MPa})\left(\frac{\pi d^2}{4}\right) - 24.19 \text{ N}$$

$$= (140 \text{ MPa})\left(\frac{\pi}{4}\right)(4.0 \text{ mm})^2 - 24.19 \text{ N}$$

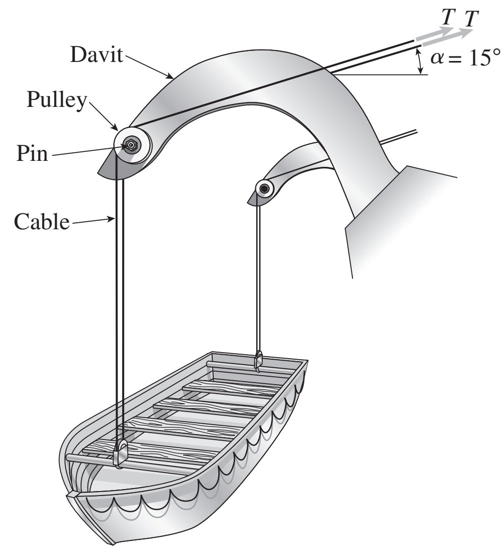
$$= 1759.3 \text{ N} - 24.2 \, N = 1735.1 \text{ N}$$

$$W_{max} = 1740 \text{ N} \quad \longleftarrow$$

Problem 1.7-7 A lifeboat hangs from two ship's davits, as shown in the figure. A pin of diameter $d = 0.80$ in. passes through each davit and supports two pulleys, one on each side of the davit.

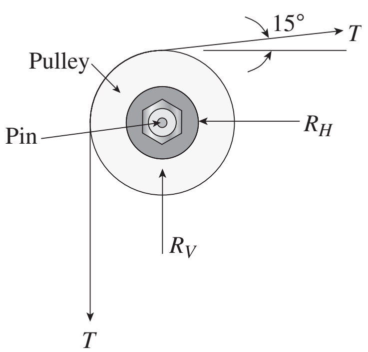
Cables attached to the lifeboat pass over the pulleys and wind around winches that raise and lower the lifeboat. The lower parts of the cables are vertical and the upper parts make an angle $\alpha = 15°$ with the horizontal. The allowable tensile force in each cable is 1800 lb, and the allowable shear stress in the pins is 4000 psi.

If the lifeboat weighs 1500 lb, what is the maximum weight that should be carried in the lifeboat?



Solution 1.7-7 Lifeboat supported by four cables

FREE-BODY DIAGRAM OF ONE PULLEY



Pin diameter $d = 0.80$ in.

T = tensile force in one cable

$T_{allow} = 1800$ lb

$\tau_{allow} = 4000$ psi

W = weight of lifeboat

$= 1500$ lb

$\Sigma F_{horiz} = 0 \qquad R_H = T \cos 15° = 0.9659T$

$\Sigma F_{vert} = 0 \qquad R_V = T - T \sin 15° = 0.7412T$

V = shear force in pin

$$V = \sqrt{(R_H)^2 + (R_V)^2} = 1.2175T$$

ALLOWABLE TENSILE FORCE IN ONE CABLE BASED UPON SHEAR IN THE PINS

$$V_{allow} = \tau_{allow} A_{pin} = (4000 \text{ psi})\left(\frac{\pi}{4}\right)(0.80 \text{ in.})^2$$

$$= 2011 \text{ lb}$$

$$V = 1.2175T \qquad T_1 = \frac{V_{allow}}{1.2175} = 1652 \text{ lb}$$

ALLOWABLE FORCE IN ONE CABLE BASED UPON TENSION IN THE CABLE

$$T_2 = T_{allow} = 1800 \text{ lb}$$

MAXIMUM WEIGHT

Shear in the pins governs.

$T_{max} = T_1 = 1652$ lb

Total tensile force in four cables

$= 4\,T_{max} = 6608$ lb

$W_{max} = 4\,T_{max} - W$

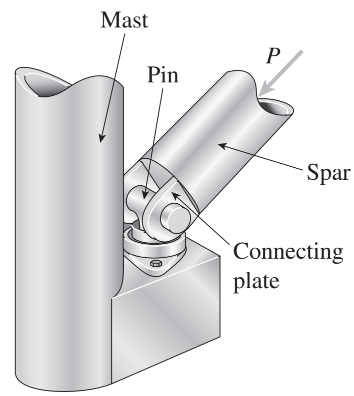
$= 6608 \text{ lb} - 1500 \text{ lb}$

$= 5110 \text{ lb}$ ⟵

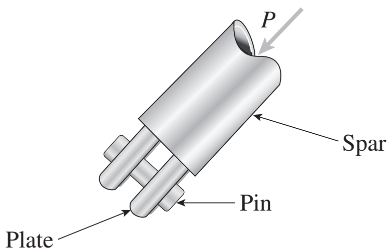
Problem 1.7-8 A ship's spar is attached at the base of a mast by a pin connection (see figure). The spar is a steel tube of outer diameter $d_2 = 80$ mm and inner diameter $d_1 = 70$ mm. The steel pin has diameter $d = 25$ mm, and the two plates connecting the spar to the pin have thickness $t = 12$ mm.

The allowable stresses are as follows: compressive stress in the spar, 70 MPa; shear stress in the pin, 45 MPa; and bearing stress between the pin and the connecting plates, 110 MPa.

Determine the allowable compressive force P_{allow} in the spar.



Solution 1.7-8 **Pin connection for a ship's spar**



Spar: $d_2 = 80$ mm

$d_1 = 70$ mm

Pin: $d = 25$ mm

Plates: $t = 12$ mm

ALLOWABLE LOAD P BASED UPON COMPRESSION IN THE SPAR

$\sigma_c = 70$ MPa

$$A_c = \frac{\pi}{4}(d_2^2 - d_1^2) = \frac{\pi}{4}[(80\text{ mm})^2 - (70\text{ mm})^2]$$

$$= 1178.1\text{ mm}^2$$

$$P_1 = \sigma_c A_c = (70\text{ MPa})(1178.1\text{ mm}^2) = 82.5\text{ kN}$$

ALLOWABLE LOAD P BASED UPON SHEAR IN THE PIN (DOUBLE SHEAR)

$\tau_{allow} = 45$ MPa

$$A_s = 2\left(\frac{\pi d^2}{4}\right) = \frac{\pi}{2}(25\text{ mm})^2 = 981.7\text{ mm}^2$$

$$P_2 = \tau_{allow} A_s = (45\text{ MPa})(981.7\text{ mm}^2) = 44.2\text{ kN}$$

ALLOWABLE LOAD P BASED UPON BEARING

$\sigma_b = 110$ MPa

$$A_b = 2dt = 2(25\text{ mm})(12\text{ mm}) = 600\text{ mm}^2$$

$$P_3 = \sigma_b A_b = (110\text{ MPa})(600\text{ mm}^2) = 66.0\text{ kN}$$

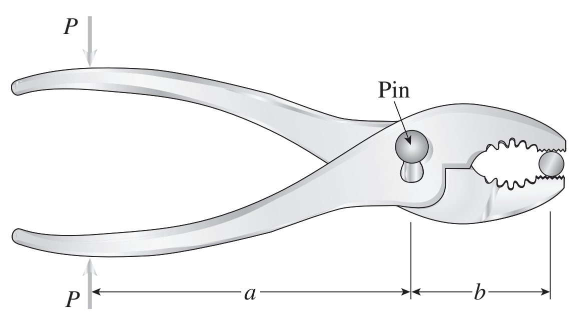
ALLOWABLE COMPRESSIVE LOAD IN THE SPAR

Shear in the pin governs.

$P_{allow} = 44.2\text{ k}N$ ⟵

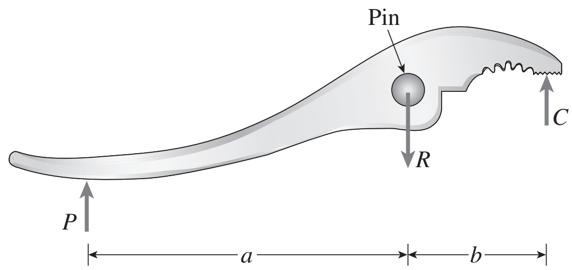
Problem 1.7-9 What is the maximum possible value of the clamping force C in the jaws of the pliers shown in the figure if $a = 3.75$ in., $b = 1.60$ in., and the ultimate shear stress in the 0.20-in. diameter pin is 50 ksi?

What is the maximum permissible value of the applied load P if a factor of safety of 3.0 with respect to failure of the pin is to be maintained?



Solution 1.7-9 Forces in pliers

FREE-BODY DIAGRAM OF ONE ARM



C = clamping force

R = reaction at pin

$a = 3.75$ in.

$b = 1.60$ in.

d = diameter of pin

$= 0.20$ in.

$\Sigma M_{pin} = 0 \quad \curvearrowleft + \curvearrowright \quad Cb - Pa = 0$

$$C = \frac{Pa}{b} \quad P = \frac{Cb}{a} \quad \frac{C}{P} = \frac{a}{b}$$

$\Sigma F_{vert} = 0 \quad \uparrow + \downarrow - \quad P + C - R = 0$

$$R = P + C = P\left(1 + \frac{a}{b}\right) = C\left(1 + \frac{b}{a}\right)$$

V = shear force in pin (single shear)

$$V = R \quad \therefore C = \frac{V}{1 + \frac{b}{a}} \quad \text{and} \quad P = \frac{V}{1 + \frac{a}{b}}$$

MAXIMUM CLAMPING FORCE C_{ult}

$\tau_{ult} = 50$ ksi

$$V_{ult} = \tau_{ult} A_{pin}$$

$$= (50 \text{ ksi})\left(\frac{\pi}{4}\right)(0.20 \text{ in.})^2$$

$$= 1571 \text{ lb}$$

$$C_{ult} = \frac{V_{ult}}{1 + \frac{b}{a}} = \frac{1571 \text{ lb}}{1 + \frac{1.60 \text{ in.}}{3.75 \text{ in.}}}$$

$$= 1100 \text{ lb} \longleftarrow$$

MAXIMUM LOAD P_{ult}

$$P_{ult} = \frac{V_{ult}}{1 + \frac{a}{b}} = \frac{1571 \text{ lb}}{1 + \frac{3.75 \text{ in.}}{1.60 \text{ in.}}} = 469.8 \text{ lb}$$

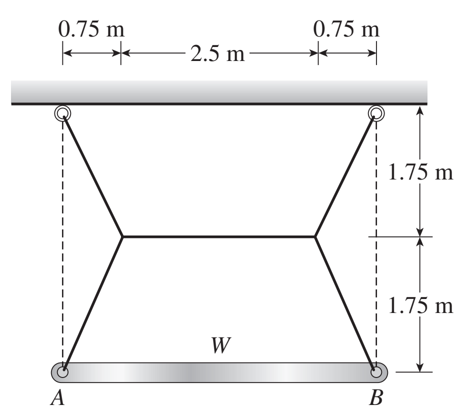
ALLOWABLE LOAD P_{allow}

$$P_{allow} = \frac{P_{ult}}{M} = \frac{469.8 \text{ lb}}{3.0}$$

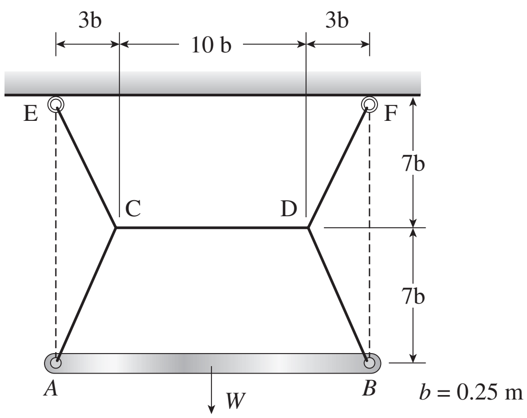
$$= 157 \text{ lb} \longleftarrow$$

Problem 1.7-10 A metal bar AB of weight W is suspended by a system of steel wires arranged as shown in the figure. The diameter of the wires is 2 mm, and the yield stress of the steel is 450 MPa.

Determine the maximum permissible weight W_{max} for a factor of safety of 1.9 with respect to yielding.

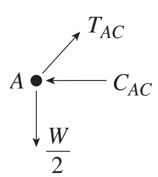


Solution 1.7-10 **Bar *AB* suspended by steel wires**



$L_{AC} = L_{EC} = \sqrt{(3b)^2 + (7b)^2} = b\sqrt{58}$

FREE-BODY DIAGRAM OF POINT *A*



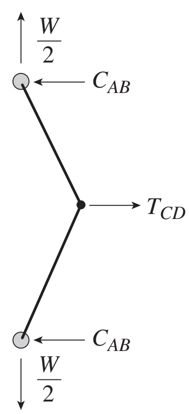
$$\Sigma F_{\text{vert}} = 0 \quad T_{AC}\left(\frac{7b}{b\sqrt{58}}\right) = \frac{W}{2}$$

$$T_{AC} = \frac{W\sqrt{58}}{14}$$

$$\Sigma F_{\text{horiz}} = 0 \quad T_{AC}\left(\frac{3b}{b\sqrt{58}}\right) = C_{AB}$$

$$C_{AB} = \frac{3W}{14}$$

FREE-BODY DIAGRAM OF WIRE *ACE*



$$\Sigma F_{\text{horiz}} = 0$$

$$T_{CD} = 2C_{AB} = \frac{3W}{7}$$

ALLOWABLE TENSILE FORCE IN A WIRE

$d = 2\,\text{mm} \qquad \sigma_y = 450\,\text{MPa} \qquad \text{F.S.} = 1.9$

$$T_{\text{allow}} = \frac{\sigma_y A}{n} = \frac{\sigma_y\left(\frac{\pi d^2}{4}\right)}{n}$$

$$= \left(\frac{450\text{ MPa}}{1.9}\right)\left(\frac{\pi}{4}\right)(2\text{ mm})^2 = 744.1\text{ N}$$

MAXIMUM TENSILE FORCES IN WIRES

$$T_{CD} = \frac{3W}{7} \qquad T_{AC} = \frac{W\sqrt{58}}{14}$$

Force in wire *AC* is larger.

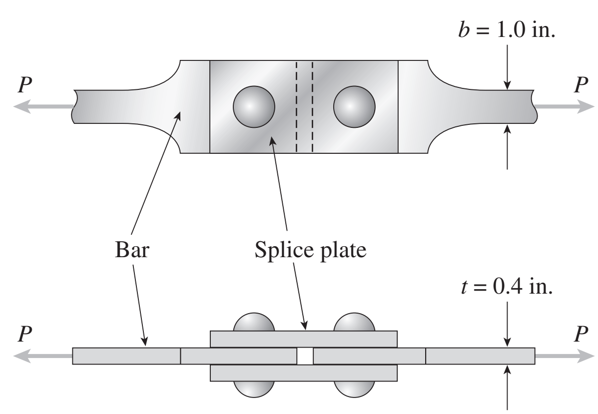
MAXIMUM ALLOWABLE WEIGHT *W*

$$W_{\max} = \frac{14\,T_{AC}}{\sqrt{58}} = \frac{14\,T_{\text{allow}}}{\sqrt{58}} = \frac{14}{\sqrt{58}}(744.1\text{ N})$$

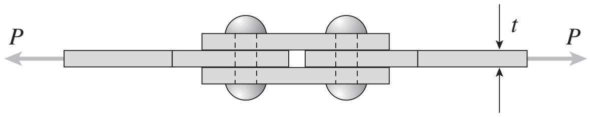
$$= 1370\text{ N} \longleftarrow$$

Problem 1.7-11 Two flat bars loaded in tension by forces *P* are spliced using two rectangular splice plates and two ⅝-in. diameter rivets (see figure). The bars have width $b = 1.0$ in. (except at the splice, where the bars are wider) and thickness $t = 0.4$ in. The bars are made of steel having an ultimate stress in tension equal to 60 ksi. The ultimate stresses in shear and bearing for the rivet steel are 25 ksi and 80 ksi, respectively.

Determine the allowable load P_{allow} if a safety factor of 2.5 is desired with respect to the ultimate load that can be carried. (Consider tension in the bars, shear in the rivets, and bearing between the rivets and the bars. Disregard friction between the plates.)



Solution 1.7-11 Splice between two flat bars



ULTIMATE LOAD BASED UPON TENSION IN THE BARS

Cross-sectional area of bars:

$A = bt$ $\quad b = 1.0\,\text{in.}$ $\quad t = 0.4\,\text{in.}$

$A = 0.40\,\text{in.}^2$

$P_1 = \sigma_{ult}\ A = (60\,\text{ksi})(0.40\,\text{in.}^2) = 24.0\,\text{k}$

ULTIMATE LOAD BASED UPON SHEAR IN THE RIVETS

Double shear $\quad d$ = diameter of rivets

$d = 5/8$ in. $\quad A_R$ = Area of rivets

$$A_R = \frac{\pi d^2}{4} = \frac{\pi}{4}\left(\frac{5}{8}\text{ in.}\right)^2 = 0.3068\text{ in.}^2$$

$$P_2 = \tau_{ult}(2A_R) = 2(25\text{ ksi})(0.3068\text{ in.}^2)$$

$$= 15.34\text{ k}$$

ULTIMATE LOAD BASED UPON BEARING

A_b = bearing area = dt

$$P_3 = \sigma_b A_b = (80\text{ ksi})\left(\frac{5}{8}\text{ in.}\right)(0.4\text{ in.}) = 20.0\text{ k}$$

ULTIMATE LOAD

Shear governs. $P_{ult} = 15.34\,\text{k}$

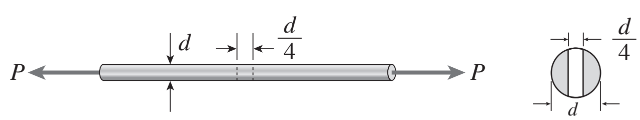
ALLOWABLE LOAD

$$P_{allow} = \frac{P_{ult}}{n} = \frac{15.34\text{ k}}{2.5} = 6.14\text{ k} \longleftarrow$$

Problem 1.7-12 A solid bar of circular cross section (diameter d) has a hole of diameter $d/4$ drilled laterally through the center of the bar (see figure). The allowable average tensile stress on the net cross section of the bar is σ_{allow}.

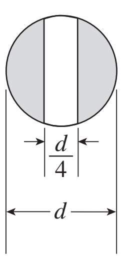
(a) Obtain a formula for the allowable load P_{allow} that the bar can carry in tension.
(b) Calculate the value of P_{allow} if the bar is made of brass with diameter $d = 40$ mm and $\sigma_{allow} = 80$ MPa.

(*Hint:* Use the formulas of Case 15, Appendix D.)



Solution 1.7-12 Bar with a hole

CROSS SECTION OF BAR



From Case 15, Appendix D:

$$A = 2r^2\left(\alpha - \frac{ab}{r^2}\right)$$

$$r = \frac{d}{2} \quad a = \frac{d}{8}$$

$$b = \sqrt{r^2 - \left(\frac{d}{8}\right)^2} = d\sqrt{\frac{15}{64}} = \frac{d}{8}\sqrt{15}$$

$$\alpha = \text{arc cos}\,\frac{d/8}{r}$$

$$= \text{arc cos}\left(\frac{1}{4}\right)$$

$$A = 2\left(\frac{d}{2}\right)^2\left[\text{arc cos}\,\frac{1}{4} - \frac{\left(\frac{d}{8}\right)\left(\frac{d}{8}\sqrt{15}\right)}{(d/2)^2}\right]$$

$$= \frac{d^2}{2}\left(\text{arc cos}\,\frac{1}{4} - \frac{\sqrt{15}}{16}\right) = 0.5380\,d^2$$

(a) ALLOWABLE LOAD IN TENSION

$$P_{allow} = \sigma_{allow} A = 0.5380 d^2\,\sigma_{allow} \quad \longleftarrow$$

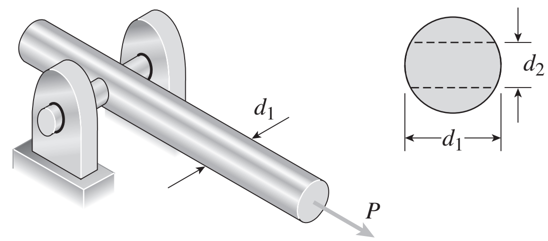
(b) SUBSTITUTE NUMERICAL VALUES

$\sigma_{allow} = 80\,\text{MPa}$ $\quad d = 40\,\text{mm}$

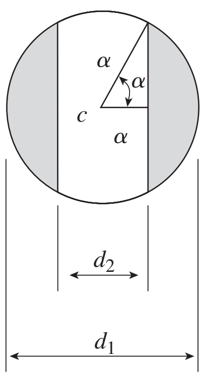
$P_{allow} = 68.9\,\text{kN} \quad \longleftarrow$

Problem 1.7-13 A solid steel bar of diameter d_1 = 2.25 in. has a hole of diameter d_2 = 1.125 in. drilled through it (see figure). A steel pin passes through the hole and is attached to supports.

Determine the maximum permissible tensile load P_{allow} in the bar if the yield stress for shear in the pin is τ_y = 17,000 psi, the yield stress for tension in the bar is σ_y 5 36,000 psi, and a factor of safety of 2.0 with respect to yielding is required. (*Hint:* Use the formulas of Case 15, Appendix D.)



Solution 1.7-13 **Bar with a hole**



$d_1 = 2.25$ in.

$d_2 = 1.125$ in.

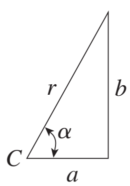
From Case 15, Appendix D:

$$A = 2r^2\left(\alpha - \frac{ab}{r^2}\right)$$

$$r = \frac{d_1}{2} = 1.125 \text{ in.}$$

$$\alpha = \text{arc cos}\frac{d_2/2}{d_1/2} = \text{arc cos}\frac{d_2}{d_1}$$

$$\frac{d_2}{d_1} = \frac{1.125 \text{ in.}}{2.25 \text{ in.}} = \frac{1}{2} \quad \alpha = \text{arc cos}\frac{1}{2} = 1.0472 \text{ rad}$$



$$a = \frac{d_2}{2} = 0.5625 \text{ in.}$$

$$b = \sqrt{r^2 - a^2} = 0.9743 \text{ in.}$$

$$A = 2r^2\left(\alpha - \frac{ab}{r^2}\right)$$

$$= 2(1.125 \text{ in.})^2$$

$$\left[1.0472 - \frac{(0.5625 \text{ in.})(0.9743 \text{ in.})}{(1.125 \text{ in.})^2}\right]$$

$$= 1.5546 \text{ in.}^2$$

Allowable load based on tension in the bar

$$P_1 = \frac{\sigma_y}{n}A = \frac{36{,}000 \text{ psi}}{2.0}(1.5546 \text{ in.}^2)$$

$$= 28.0 \text{ k}$$

Allowable load based on shear in the pin

Double shear

$$A_s = 2A_{pin} = 2\left(\frac{\pi d_2^2}{4}\right) = \frac{\pi}{2}(1.125 \text{ in.})^2$$

$$= 1.9880 \text{ in.}^2$$

$$P_2 = \frac{\tau_y}{n}A_s = \frac{17{,}000 \text{ psi}}{2.0}(1.9880 \text{ in.})^2$$

$$= 16.9 \text{ k}$$

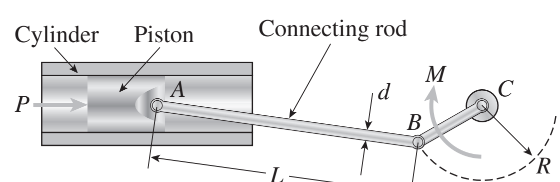
Allowable load

Shear in the pin governs.

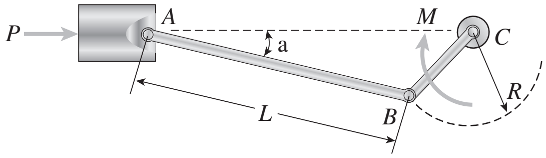
$P_{allow} = 16.9$ k ⟵

Problem 1.7-14 The piston in an engine is attached to a connecting rod *AB*, which in turn is connected to a crank arm *BC* (see figure). The piston slides without friction in a cylinder and is subjected to a force *P* (assumed to be constant) while moving to the right in the figure. The connecting rod, which has diameter *d* and length *L*, is attached at both ends by pins. The crank arm rotates about the axle at *C* with the pin at *B* moving in a circle of radius *R*. The axle at *C*, which is supported by bearings, exerts a resisting moment *M* against the crank arm.

(a) Obtain a formula for the maximum permissible force P_{allow} based upon an allowable compressive stress σ_c in the connecting rod.
(b) Calculate the force P_{allow} for the following data: σ_c = 160 MPa, d = 9.00 mm, and $R = 0.28L$.

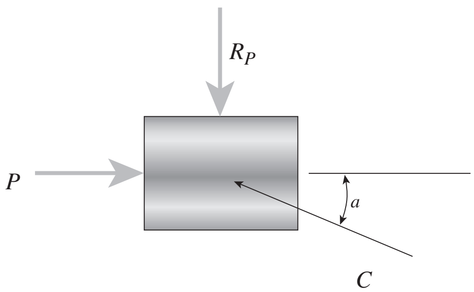


Solution 1.7-14 **Piston and connecting rod**



d = diameter of rod AB

FREE-BODY DIAGRAM OF PISTON



P = applied force (constant)

C = compressive force in connecting rod

R_P = resultant of reaction forces between cylinder and piston (no friction)

$\Sigma F_{horiz} = 0 \quad \overset{+}{\rightarrow} \overset{-}{\leftarrow} \quad P - C\cos\alpha = 0$

$P = C\cos\alpha$

MAXIMUM COMPRESSIVE FORCE C IN CONNECTING ROD

$$C_{max} = \sigma_c A_c$$

in which A_c = area of connecting rod

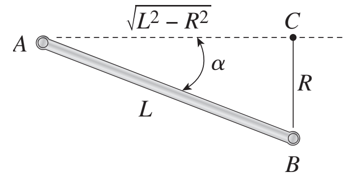
$$A_c = \frac{\pi d^2}{4}$$

MAXIMUM ALLOWABLE FORCE P

$$P = C_{max}\cos\alpha$$
$$= \sigma_c A_c \cos\alpha$$

The maximum allowable force P occurs when $\cos\alpha$ has its smallest value, which means that α has its largest value.

LARGEST VALUE OF α



The largest value of α occurs when point B is the farthest distance from line AC. The farthest distance is the radius R of the crank arm.

Therefore,

$\overline{BC} = R$

Also, $\overline{AC} = \sqrt{L^2 - R^2}$

$$\cos\alpha = \frac{\sqrt{L^2 - R^2}}{L} = \sqrt{1 - \left(\frac{R}{L}\right)^2}$$

(a) MAXIMUM ALLOWABLE FORCE P

$$P_{allow} = \sigma_c A_c \cos\alpha$$
$$= \sigma_c\left(\frac{\pi d^2}{4}\right)\sqrt{1 - \left(\frac{R}{L}\right)^2} \quad \longleftarrow$$

(b) SUBSTITUTE NUMERICAL VALUES

$\sigma_c = 160$ MPa $\qquad d = 9.00$ mm

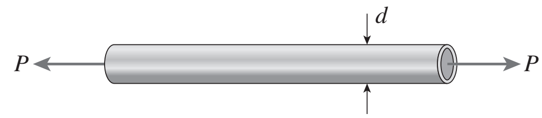
$R = 0.28$ L $\qquad R/L = 0.28$

$P_{allow} = 9.77$ kN $\quad \longleftarrow$

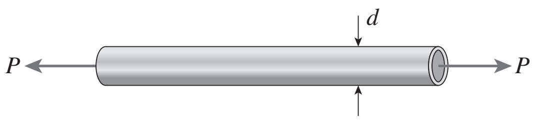
Design for Axial Loads and Direct Shear

Problem 1.8-1 An aluminum tube is required to transmit an axial tensile force $P = 34$ k (see figure). The thickness of the wall of the tube is to be 0.375 in.

What is the minimum required outer diameter d_{min} if the allowable tensile stress is 9000 psi?



Solution 1.8-1 Aluminum tube in tension



$P = 34\,\text{k}$

$t = 0.375\,\text{in.}$

$\sigma_{allow} = 9000\,\text{psi}$

$$A = \frac{\pi}{4}[d^2 - (d - 2t)^2] = \frac{\pi}{4}(4t)(d - t)$$

$$= \pi t(d - t)$$

$$P = \sigma_{allow} A = \pi t(d - t)\sigma_{allow}$$

SOLVE FOR d:

$$d = \frac{P}{\pi t \sigma_{allow}} + t$$

SUBSTITUTE NUMERICAL VALUES:

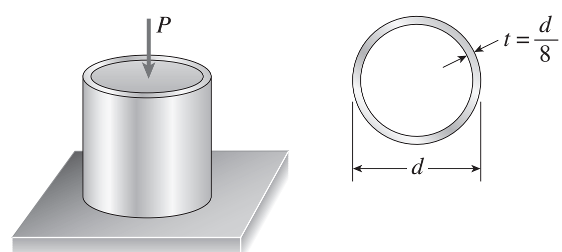
$$d_{min} = \frac{34\text{ k}}{\pi(0.375\text{ in.})(9000\text{ psi})} + 0.375\text{ in.}$$

$$= 3.207\text{ in.} + 0.375\text{ in.}$$

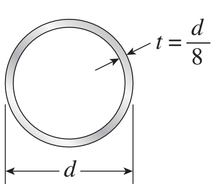
$$d_{min} = 3.58\text{ in.} \longleftarrow$$

Problem 1.8-2 A steel pipe having yield stress $\sigma_y = 270$ MPa is to carry an axial compressive load $P = 1200$ kN (see figure). A factor of safety of 1.8 against yielding is to be used.

If the thickness t of the pipe is to be one-eighth of its outer diameter, what is the minimum required outer diameter d_{min}?



Solution 1.8-2 Steel pipe in compression



$P = 1200\,\text{kN}$

$\sigma_Y = 270\,\text{MPa}$

$n = 1.8$

$\sigma_{allow} = 150\,\text{MPa}$

$$A = \frac{\pi}{4}\left[d^2 - \left(d - \frac{d}{4}\right)^2\right] = \frac{7\pi d^2}{64}$$

$$P = \sigma_{allow} \quad A = \frac{7\pi d^2}{64}\sigma_{allow}$$

SOLVE FOR d:

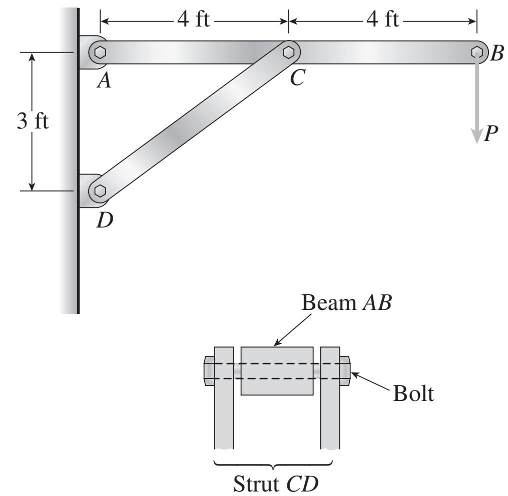
$$d^2 = \frac{64\,P}{7\pi\sigma_{allow}} \qquad d = 8\sqrt{\frac{P}{7\pi\sigma_{allow}}}$$

SUBSTITUTE NUMERICAL VALUES:

$$d_{min} = 8\sqrt{\frac{1200\text{ kN}}{7\pi\,(150\text{ MPa})}} = 153\text{ mm} \longleftarrow$$

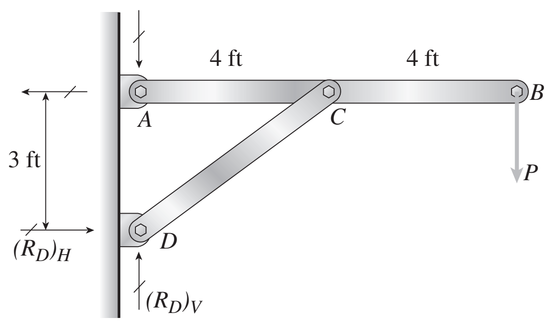
Problem 1.8-3 A horizontal beam *AB* supported by an inclined strut *CD* carries a load $P = 2500$ lb at the position shown in the figure. The strut, which consists of two bars, is connected to the beam by a bolt passing through the three bars meeting at joint *C*.

If the allowable shear stress in the bolt is 14,000 psi, what is the minimum required diameter d_{min} of the bolt?



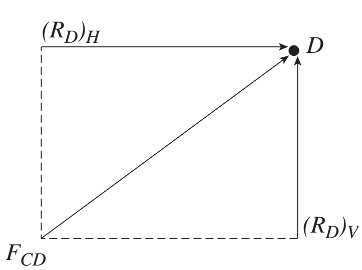
Solution 1.8-3 **Beam *ACB* supported by a strut *CD***

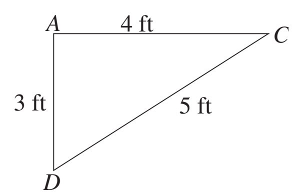
FREE-BODY DIAGRAM



$\Sigma M_A = 0$ ↺ $- P(8 \text{ ft}) + (R_D)_H (3 \text{ ft}) = 0$

$$(R_D)_H = \frac{8}{3} P$$





REACTION AT JOINT *D*

F_{CD} = compressive force in strut

$$F_{CD} = (R_D)_H\left(\frac{5}{4}\right) = \left(\frac{5}{4}\right)\left(\frac{8P}{3}\right) = \frac{10P}{3}$$

SHEAR FORCE ACTING ON BOLT

$$V = \frac{F_{CD}}{2} = \frac{5P}{3}$$

REQUIRED AREA AND DIAMETER OF BOLT

$$A = \frac{V}{\tau_{allow}} = \frac{5P}{3\tau_{allow}} \qquad A = \frac{\pi d^2}{4} \qquad d^2 = \frac{20P}{3\pi\tau_{allow}}$$

SUBSTITUTE NUMERICAL VALUES:

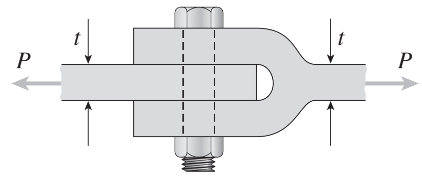
$P = 2500$ lb $\qquad \tau_{allow} = 14{,}000$ psi

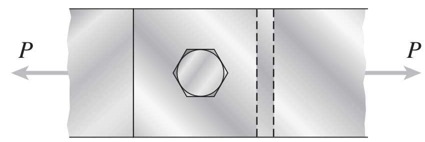
$d^2 = 0.3789 \text{ in.}^2$

$d_{min} = 0.616$ in. ⟵

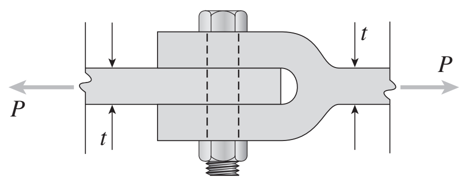
Problem 1.8-4 Two bars of rectangular cross section (thickness $t = 15$ mm) are connected by a bolt in the manner shown in the figure. The allowable shear stress in the bolt is 90 MPa and the allowable bearing stress between the bolt and the bars is 150 MPa.

If the tensile load $P = 31$ kN, what is the minimum required diameter d_{min} of the bolt?





Solution 1.8-4 **Bolted connection**



One bolt in double shear.

$P = 31\,\text{kN}$

$\tau_{allow} = 90\,\text{MPa}$

$\sigma_b = 150\,\text{MPa}$

$t = 15\,\text{mm}$

Find minimum diameter of bolt.

Based upon shear in the bolt.

$$A_{bolt} = \frac{P}{2\tau_{allow}} \qquad \frac{\pi d^2}{4} = \frac{P}{2\tau_{allow}}$$

$$d^2 = \frac{2P}{\pi\tau_{allow}}$$

$$d_1 = \sqrt{\frac{2P}{\pi\tau_{allow}}} = \sqrt{\frac{2(31\text{ kN})}{\pi(90\text{ MPa})}}$$

$$= 14.8\text{ mm}$$

Based upon bearing between plate and bolt

$$A_{bearing} = \frac{P}{\sigma_b} \qquad dt = \frac{P}{\sigma_b}$$

$$d = \frac{P}{t\sigma_b} \qquad d_2 = \frac{31\text{ kN}}{(15\text{ mm})(150\text{ MPa})} = 13.8\text{ mm}$$

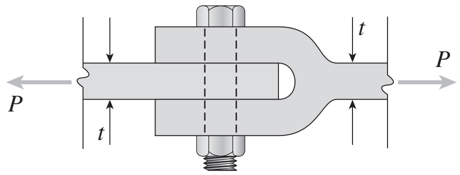
Minimum diameter of bolt

Shear governs.

$d_{min} = 14.8\text{ mm} \longleftarrow$

Problem 1.8-5 Solve the preceding problem if the bars have thickness $t = \frac{5}{16}$ in., the allowable shear stress is 12,000 psi, the allowable bearing stress is 20,000 psi, and the load $P = 1800$ lb.

Solution 1.8-5 Bolted connection



One bolt in double shear.

$P = 1800$ lb

$\tau_{allow} = 12,000$ psi

$\sigma_b = 20,000$ psi

$t = \frac{5}{16}$ in.

Find minimum diameter of bolt.

BASED UPON SHEAR IN THE BOLT

$$A_{bolt} = \frac{P}{2\tau_{allow}} \qquad \frac{\pi d^2}{4} = \frac{P}{2\tau_{allow}}$$

$$d^2 = \frac{2P}{\pi\tau_{allow}}$$

$$d_1 = \sqrt{\frac{2P}{\pi\tau_{allow}}} = \sqrt{\frac{2(1800 \text{ lb})}{\pi(12,000 \text{ psi})}} = 0.309 \text{ in.}$$

BASED UPON BEARING BETWEEN PLATE AND BOLT

$$A_{bearing} = \frac{P}{\sigma_b} \qquad dt = \frac{P}{\sigma_b}$$

$$d = \frac{P}{t\sigma_b} \qquad d_2 = \frac{1800 \text{ lb}}{(\frac{5}{16} \text{ in.})(20,000 \text{ psi})} = 0.288 \text{ in.}$$

MINIMUM DIAMETER OF BOLT

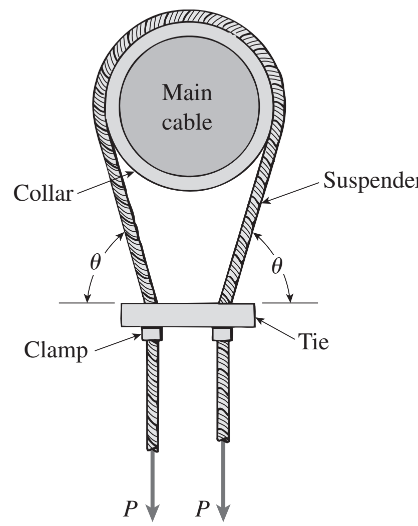
Shear governs.

$d_{min} = 0.309$ in. ⟵

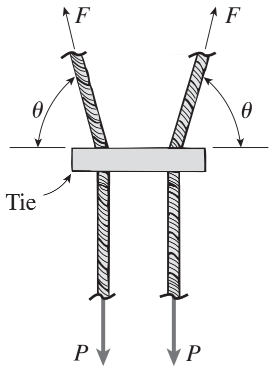
Problem 1.8-6 A suspender on a suspension bridge consists of a cable that passes over the main cable (see figure) and supports the bridge deck, which is far below. The suspender is held in position by a metal tie that is prevented from sliding downward by clamps around the suspender cable.

Let P represent the load in each part of the suspender cable, and let θ represent the angle of the suspender cable just above the tie. Finally, let σ_{allow} represent the allowable tensile stress in the metal tie.

(a) Obtain a formula for the minimum required cross-sectional area of the tie.
(b) Calculate the minimum area if $P = 130$ kN, $\theta = 75°$, and $\sigma_{allow} = 80$ MPa.



Solution 1.8-6 Suspender tie on a suspension bridge



F = tensile force in cable above tie

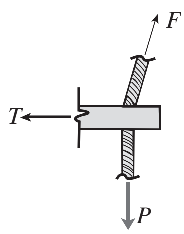
P = tensile force in cable below tie

σ_{allow} = allowable tensile stress in the tie

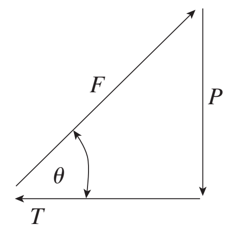
FREE-BODY DIAGRAM OF HALF THE TIE

Note: Include a small amount of the cable in the free-body diagram

T = tensile force in the tie



FORCE TRIANGLE



$$\cot\theta = \frac{T}{P}$$

$$T = P\cot\theta$$

(a) MINIMUM REQUIRED AREA OF TIE

$$A_{min} = \frac{T}{\sigma_{allow}} = \frac{P\cot\theta}{\sigma_{allow}} \longleftarrow$$

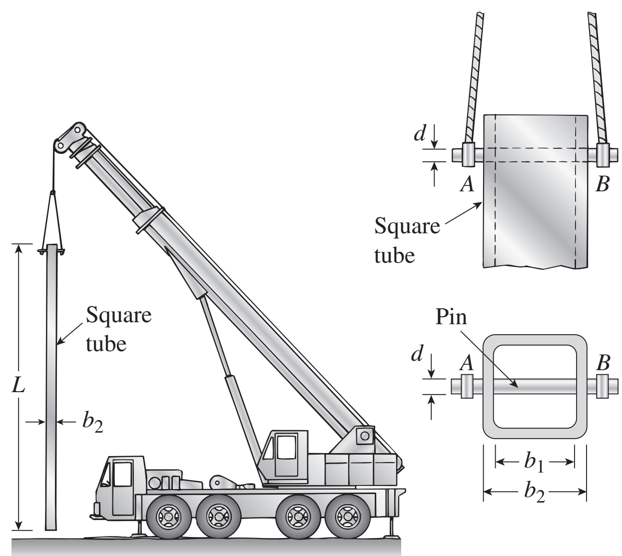
(b) SUBSTITUTE NUMERICAL VALUES:

$P = 130$ kN $\theta = 75°$ $\sigma_{allow} = 80$ MPa

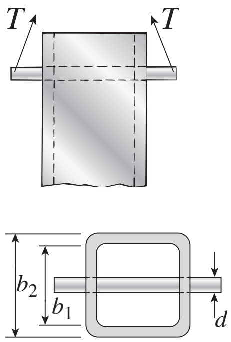
$A_{min} = 435\ \text{mm}^2 \longleftarrow$

Problem 1.8-7 A square steel tube of length $L = 20$ ft and width $b_2 = 10.0$ in. is hoisted by a crane (see figure). The tube hangs from a pin of diameter d that is held by the cables at points A and B. The cross section is a hollow square with inner dimension $b_1 = 8.5$ in. and outer dimension $b_2 = 10.0$ in. The allowable shear stress in the pin is 8,700 psi, and the allowable bearing stress between the pin and the tube is 13,000 psi.

Determine the minimum diameter of the pin in order to support the weight of the tube. (*Note:* Disregard the rounded corners of the tube when calculating its weight.)



Solution 1.8-7 **Tube hoisted by a crane**



T = tensile force in cable

W = weight of steel tube

d = diameter of pin

b_1 = inner dimension of tube = 8.5 in.

b_2 = outer dimension of tube = 10.0 in.

L = Length of tube = 20 ft

$\tau_{allow} = 8{,}700$ psi

$\sigma_b = 13{,}000$ psi

Weight of tube

γ_s = weight density of steel = 490 lb/ft^3

A = area of tube

$$= b_2^2 - b_1^2 = (10.0 \text{ in.})^2 - (8.5 \text{ in.})^2 = 27.75 \text{ in.}$$

$$W = \gamma_s AL = (490 \text{ lb/ft}^3)(27.75 \text{ in.}^2)\left(\frac{1}{144}\frac{\text{ft}^2}{\text{in.}}\right)(20 \text{ ft}) = 1{,}889 \text{ lb}$$

Diameter of pin based upon shear

Double shear. $2\,\tau_{allow}\,A_{pin} = W$

$$2(8{,}700 \text{ psi})\left(\frac{\pi d^2}{4}\right) = 1889 \text{ lb}$$

$d^2 = 0.1382 \text{ in.}^2$ $\qquad d_1 = 0.372$ in.

Diameter of pin based upon bearing

$\sigma_b(b_2 - b_1)\,d = W$

$(13{,}000 \text{ psi})(10.0 \text{ in.} - 8.5 \text{ in.})\,d = 1{,}889$ lb

$d_2 = 0.097$ in.

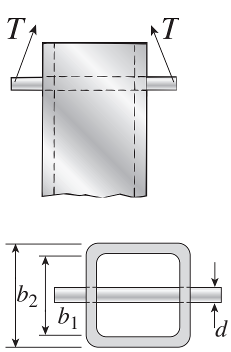
Minimum diameter of pin

Shear governs.

$d_{min} = 0.372$ in. ⟵

Problem 1.8-8 Solve the preceding problem if the length L of the tube is 6.0 m, the outer width is $b_2 = 250$ mm, the inner dimension is $b_1 = 210$ mm, the allowable shear stress in the pin is 60 MPa, and the allowable bearing stress is 90 MPa.

Solution 1.8-8 **Tube hoisted by a crane**



T = tensile force in cable

W = weight of steel tube

d = diameter of pin

b_1 = inner dimension of tube = 210 mm

b_2 = outer dimension of tube = 250 mm

L = length of tube = 6.0 m

$\tau_{allow} = 60$ MPa

$\sigma_b = 90$ MPa

Weight of tube

γ_s = Weight density of steel = 77.0 kN/m^3

A = area of tube

$b_2^2 - b_1^2 = 18{,}400 \text{ mm}^2$

$$W = \gamma_s AL = (77.0 \text{ kN/m}^3)(18{,}400 \text{ mm}^2)(6.0 \text{ m}) = 8.501 \text{ kN}$$

Diameter of pin based upon shear

Double shear. $2\tau_{allow}A_{pin} = W$

$$2(60 \text{ MPa})\left(\frac{\pi}{4}\right)d^2 = 8.501 \text{ kN} \qquad d^2 = 90.20 \text{ mm}^2$$

$d_1 = 9.497$ mm

Diameter of pin based upon bearing

$\sigma_b(b_2 - b_1)d = W$

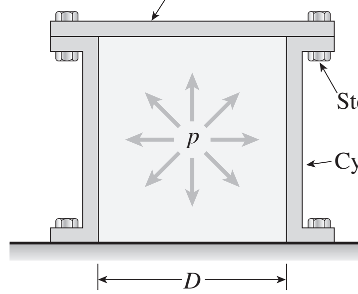
$(90 \text{ MPa})(40 \text{ mm})d = 8.501 \text{ kN} \qquad d_2 = 2.361$ mm

Minimum diameter of pin

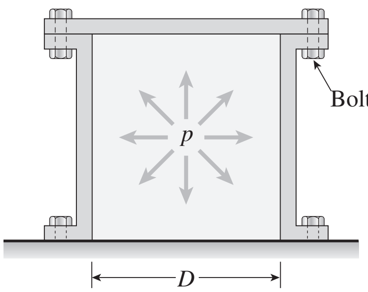
Shear governs. $\qquad d_{min} = 9.50$ mm ⟵

Problem 1.8-9 A pressurized circular cylinder has a sealed cover plate fastened with steel bolts (see figure). The pressure p of the gas in the cylinder is 290 psi, the inside diameter D of the cylinder is 10.0 in., and the diameter d_B of the bolts is 0.50 in.

If the allowable tensile stress in the bolts is 10,000 psi, find the number n of bolts needed to fasten the cover.



Solution 1.8-9 **Pressurized cylinder**



$p = 290$ psi

$D = 10.0$ in.

$d_b = 0.50$ in.

$\sigma_{allow} = 10{,}000$ psi

n = number of bolts

F = total force acting on the cover plate from the internal pressure

$$F = p\left(\frac{\pi D^2}{4}\right)$$

Number of bolts

p = tensile force in one bolt

$$p = \frac{F}{n} = \frac{\pi p D^2}{4n}$$

$$A_b = \text{area of one bolt} = \frac{\pi}{4} d_b^2$$

$$p = \sigma_{allow} A_b$$

$$\sigma_{allow} = \frac{p}{A_b} = \frac{\pi p D^2}{(4n)(\frac{\pi}{4})d_b^2} = \frac{pD^2}{nd_b^2}$$

$$n = \frac{pD^2}{d_b^2 \sigma_{allow}}$$

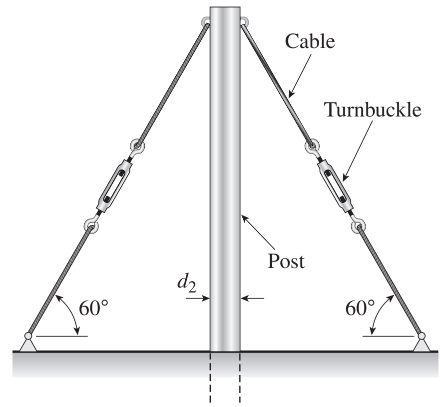
Substitute numerical values:

$$n = \frac{(290 \text{ psi})(10 \text{ in.})^2}{(0.5 \text{ in.})^2(10{,}000 \text{ psi})} = 11.6$$

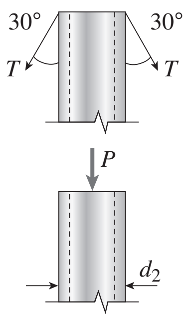
Use 12 bolts ⟵

Problem 1.8-10 A tubular post of outer diameter d_2 is guyed by two cables fitted with turnbuckles (see figure). The cables are tightened by rotating the turnbuckles, thus producing tension in the cables and compression in the post. Both cables are tightened to a tensile force of 110 kN. Also, the angle between the cables and the ground is 60°, and the allowable compressive stress in the post is $\sigma_c = 35$ MPa.

If the wall thickness of the post is 15 mm, what is the minimum permissible value of the outer diameter d_2?



Solution 1.8-10 Tubular post with guy cables



d_2 = outer diameter

d_1 = inner diameter

t = wall thickness = 15 mm

T = tensile force in a cable = 110 kN

σ_{allow} = 35 MPa

P = compressive force in post $= 2T\cos 30°$

REQUIRED AREA OF POST

$$A = \frac{P}{\sigma_{allow}} = \frac{2T\cos 30°}{\sigma_{allow}}$$

AREA OF POST

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = \frac{\pi}{4}[d_2^2 - (d_2 - 2t)^2]$$

$$= \pi t(d_2 - t)$$

EQUATE AREAS AND SOLVE FOR d_2:

$$\frac{2T\cos 30°}{\sigma_{allow}} = \pi t(d_2 - t)$$

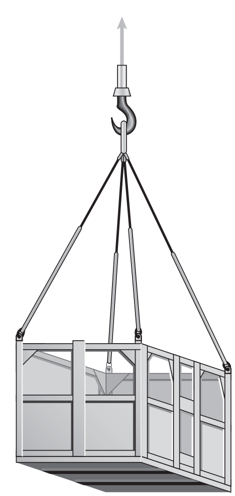
$$d_2 = \frac{2T\cos 30°}{\pi t \sigma_{allow}} + t \longleftarrow$$

SUBSTITUTE NUMERICAL VALUES:

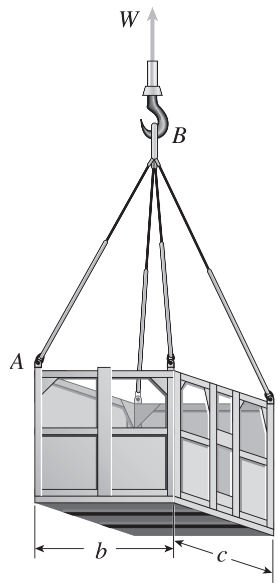
$$(d_2)_{min} = 131 \text{ mm} \longleftarrow$$

Problem 1.8-11 A cage for transporting workers and supplies on a construction site is hoisted by a crane (see figure). The floor of the cage is rectangular with dimensions 6 ft by 8 ft. Each of the four lifting cables is attached to a corner of the cage and is 13 ft long. The weight of the cage and its contents is limited by regulations to 9600 lb.

Determine the required cross-sectional area A_C of a cable if the breaking stress of a cable is 91 ksi and a factor of safety of 3.5 with respect to failure is desired.



Solution 1.8-11 Cage hoisted by a crane



Dimensions of cage:

$b = 6$ ft $\qquad c = 8$ ft

Length of a cable: $L = 13$ ft

Weight of cage and contents:

$W = 9600$ lb

Breaking stress of a cable:

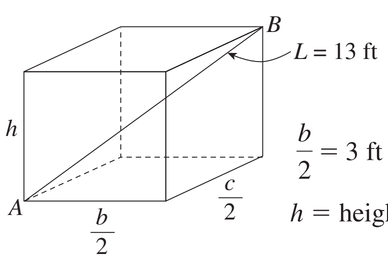
$\sigma_{ult} = 91$ ksi

Factor of safety: $n = 3.5$

$$\sigma_{allow} = \frac{\sigma_{ult}}{n} = \frac{91 \text{ ksi}}{3.5} = 26{,}000 \text{ psi}$$

GEOMETRY OF ONE CABLE (CABLE AB)

Point B is above the midpoint of the cage



$\frac{b}{2} = 3$ ft $\qquad \frac{c}{2} = 4$ ft

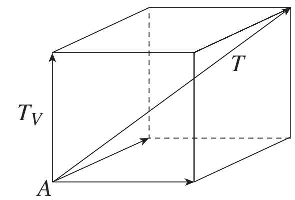
h = height from A to B

From geometry: $\quad L^2 = \left(\frac{b}{2}\right)^2 + \left(\frac{c}{2}\right)^2 + h^2$

$(13 \text{ ft})^2 = (3 \text{ ft})^2 + (4 \text{ ft})^2 + h^2$

Solving, $h = 12$ ft

FORCE IN A CABLE



T = force in one cable (cable AB)

T_V = vertical component of T

(Each cable carries the same load.)

$$\therefore T_V = \frac{W}{4} = \frac{9600 \text{ lb}}{4} = 2400 \text{ lb}$$

$$\frac{T}{T_V} = \frac{L}{h} = \frac{13 \text{ ft}}{12 \text{ ft}}$$

$$T = \frac{13}{12} T_V = 2600 \text{ lb}$$

REQUIRED AREA OF CABLE

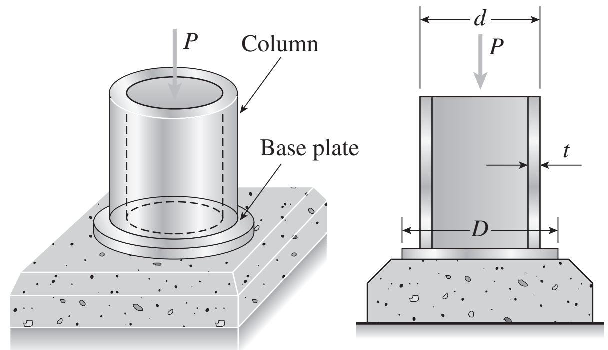
$$A_c = \frac{T}{\sigma_{allow}} = \frac{2{,}600 \text{ lb}}{26{,}000 \text{ psi}} = 0.100 \text{ in.}^2 \longleftarrow$$

(Note: The diameter of the cable cannot be calculated from the area A_c, because a cable does not have a solid circular cross section. A cable consists of several strands wound together. For details, see Section 2.2.)

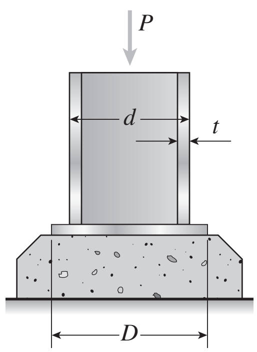
Problem 1.8-12 A steel column of hollow circular cross section is supported on a circular steel base plate and a concrete pedestal (see figure). The column has outside diameter d = 250 mm and supports a load P = 750 kN.

(a) If the allowable stress in the column is 55 MPa, what is the minimum required thickness t? Based upon your result, select a thickness for the column. (Select a thickness that is an even integer, such as 10, 12, 14, . . ., in units of millimeters.)

(b) If the allowable bearing stress on the concrete pedestal is 11.5 MPa, what is the minimum required diameter D of the base plate if it is designed for the allowable load P_{allow} that the column with the selected thickness can support?



Solution 1.8-12 **Hollow circular column**



d = 250 mm

P = 750 kN

σ_{allow} = 55 MPa (compression in column)

t = thickness of column

D = diameter of base plate

σ_b = 11.5 MPa (allowable pressure on concrete)

(a) THICKNESS t OF THE COLUMN

$$A = \frac{P}{\sigma_{allow}} \quad A = \frac{\pi d^2}{4} - \frac{\pi}{4}(d - 2t)^2$$

$$= \frac{\pi}{4}(4t)(d - t) = \pi t(d - t)$$

$$\pi t(d - t) = \frac{P}{\sigma_{allow}}$$

$$\pi t^2 - \pi t d + \frac{P}{\sigma_{allow}} = 0$$

$$t^2 - dt + \frac{P}{\pi \sigma_{allow}} = 0 \qquad \text{(Eq. 1)}$$

SUBSTITUTE NUMERICAL VALUES IN EQ. (1):

$$t^2 - 250\,t + \frac{(750 \times 10^3\text{ N})}{\pi(55\text{ N/mm}^2)} = 0$$

(Note: In this eq., t has units of mm.)

$$t^2 - 250t + 4{,}340.6 = 0$$

Solve the quadratic eq. for t:

t = 18.77 mm $\quad t_{min}$ = 18.8 mm ⟵

Use t = 20 mm ⟵

(b) DIAMETER D OF THE BASE PLATE

For the column, $\quad P_{allow} = \sigma_{allow} A$

where A is the area of the column with t = 20 mm.

$A = \pi t(d - t) \quad P_{allow} = \sigma_{allow}\,\pi t(d - t)$

$$\text{Area of base plate} = \frac{\pi D^2}{4} = \frac{P_{allow}}{\sigma_b}$$

$$\frac{\pi D^2}{4} = \frac{\sigma_{allow} \pi t(d - t)}{\sigma_b}$$

$$D^2 = \frac{4\sigma_{allow} t(d - t)}{\sigma_b}$$

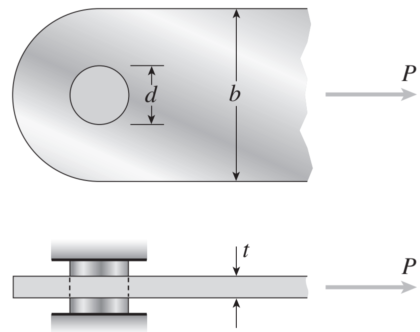
$$= \frac{4(55\text{ MPa})(20\text{ mm})(230\text{ mm})}{11.5\text{ MPa}}$$

D^2 = 88,000 mm^2 $\quad D$ = 296.6 mm

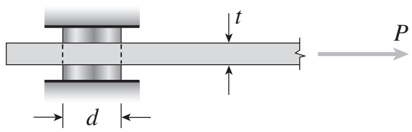
D_{min} = 297 mm ⟵

Problem 1.8-13 A bar of rectangular cross section is subjected to an axial load P (see figure). The bar has width $b = 2.0$ in. and thickness $t = 0.25$ in. A hole of diameter d is drilled through the bar to provide for a pin support. The allowable tensile stress on the net cross section of the bar is 20 ksi, and the allowable shear stress in the pin is 11.5 ksi.

(a) Determine the pin diameter d_m for which the load P will be a maximum.
(b) Determine the corresponding value P_{max} of the load.



Solution 1.8-13 **Bar with pin connection**



Width of bar $b = 2$ in.

Thickness $t = 0.25$ in.

$\sigma_{allow} = 20$ ksi

$\tau_{allow} = 11.5$ ksi

d = diameter of pin (inches)

P = axial load (pounds)

ALLOWABLE LOAD BASED UPON TENSION IN BAR

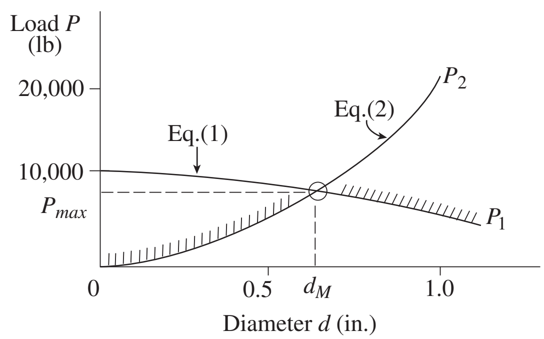
$$P_1 = \sigma_{allow} A_{net} = \sigma_{allow}(b - d)t$$
$$= (20{,}000 \text{ psi})(2 \text{ in.} - d)(0.25 \text{ in.})$$
$$= 5{,}000(2 - d) = 10{,}000 - 5{,}000d \qquad \text{Eq. (1)}$$

ALLOWABLE LOAD BASED UPON SHEAR IN PIN

Double shear

$$P_2 = 2\tau_{allow}\left(\frac{\pi d^2}{4}\right) = \tau_{allow}\left(\frac{\pi d^2}{2}\right)$$
$$= (11{,}500 \text{ psi})\left(\frac{\pi d^2}{2}\right) = 18{,}064d^2 \qquad \text{Eq. (2)}$$

GRAPH OF EQS. (1) AND (2)



(a) MAXIMUM LOAD OCCURS WHEN $P_1 = P_2$

$$10{,}000 - 5{,}000d = 18{,}064d^2$$
$$\text{or } 18{,}064d^2 + 5{,}000d - 10{,}000 = 0$$

Solve quadratic equation:

$d = 0.6184$ in. $\quad d_m = 0.618$ in. ⟵

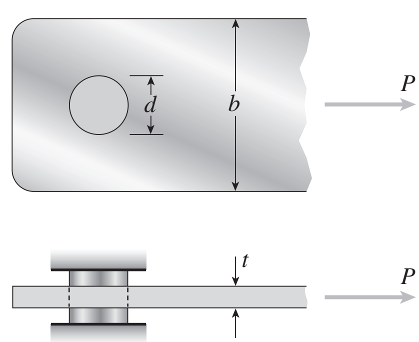
(b) MAXIMUM LOAD

Substitute $d = 0.6184$ in. into Eq. (1) or Eq. (2):

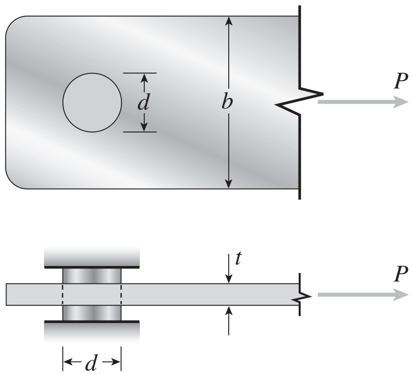
$P_{max} = 6910$ lb ⟵

Problem 1.8-14 A flat bar of width $b = 60$ mm and thickness $t = 10$ mm is loaded in tension by a force P (see figure). The bar is attached to a support by a pin of diameter d that passes through a hole of the same size in the bar. The allowable tensile stress on the net cross section of the bar is $\sigma_T = 140$ MPa, the allowable shear stress in the pin is $\tau_S = 80$ MPa, and the allowable bearing stress between the pin and the bar is $\sigma_B = 200$ MPa.

(a) Determine the pin diameter d_m for which the load P will be a maximum.
(b) Determine the corresponding value P_{max} of the load.



Solution 1.8-14 Bar with a pin connection



$b = 60\,\text{mm}$

$t = 10\,\text{mm}$

d = diameter of hole and pin

$\sigma_T = 140\,\text{MPa}$

$\tau_s = 80\,\text{MPa}$

$\sigma_B = 200\,\text{MPa}$

UNITS USED IN THE FOLLOWING CALCULATIONS:

P is in kN

σ and τ are in N/mm^2 (same as MPa)

b, t, and d are in mm

TENSION IN THE BAR

$$P_T = \sigma_T(\text{Net area}) = \sigma_t(t)(b - d)$$

$$= (140\text{ MPa})(10\text{ mm})(60\text{ mm} - d)\left(\frac{1}{1000}\right)$$

$$= 1.40\,(60 - d) \qquad \text{(Eq. 1)}$$

SHEAR IN THE PIN

$$P_s = 2\tau_s A_{\text{pin}} = 2\tau_s\left(\frac{\pi d^2}{4}\right)$$

$$= 2(80\text{ MPa})\left(\frac{\pi}{4}\right)(d^2)\left(\frac{1}{1000}\right)$$

$$= 0.040\,\pi d^2 = 0.12566\,d^2 \qquad \text{(Eq. 2)}$$

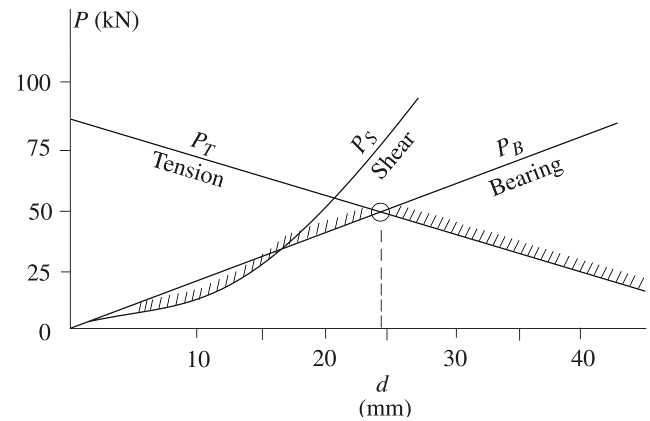
BEARING BETWEEN PIN AND BAR

$$P_B = \sigma_B\, td$$

$$= (200\text{ MPa})(10\text{ mm})(d)\left(\frac{1}{1000}\right)$$

$$= 2.0\,d \qquad \text{(Eq. 3)}$$

GRAPH OF EQS. (1), (2), AND (3)



(a) MAXIMUM ALLOWABLE LOAD P

$P_T = P_B$ or $1.40(60 - d) = 2.0\,d$

Solving, $d_m = \dfrac{84.0}{3.4}$ mm $= 24.7$ mm ⟵

(b) LOAD $P_{\max}$

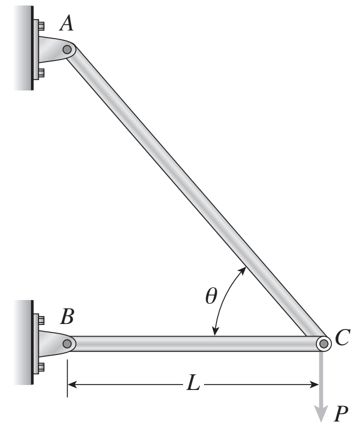
Substitute d into Eq. (1) or Eq. (3):

$P_{\max} = 49.4$ kN ⟵

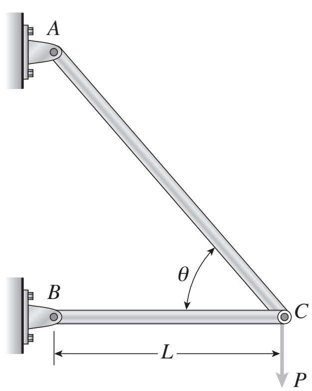
Problem 1.8-15 Two bars AC and BC of the same material support a vertical load P (see figure). The length L of the horizontal bar is fixed, but the angle θ can be varied by moving support A vertically and changing the length of bar AC to correspond with the new position of support A. The allowable stresses in the bars are the same in tension and compression.

We observe that when the angle θ is reduced, bar AC becomes shorter but the cross-sectional areas of both bars increase (because the axial forces are larger). The opposite effects occur if the angle θ is increased. Thus, we see that the weight of the structure (which is proportional to the volume) depends upon the angle θ.

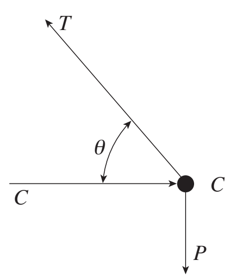
Determine the angle θ so that the structure has minimum weight without exceeding the allowable stresses in the bars. (*Note*: The weights of the bars are very small compared to the force P and may be disregarded.)



Solution 1.8-15 Two bars supporting a load *P*



Joint C



T = tensile force in bar AC

c = Compressive force in bar BC

$$\Sigma F_{vert} = 0 \qquad T = \frac{P}{\sin\theta}$$

$$\Sigma F_{horiz} = 0 \qquad C = \frac{P}{\tan\theta}$$

AREAS OF BARS

$$A_{ac} = \frac{T}{\sigma_{allow}} = \frac{P}{\sigma_{allow}\sin\theta}$$

$$A_{bc} = \frac{c}{\sigma_{allow}} = \frac{c}{\sigma_{allow}\tan\theta}$$

LENGTHS OF BARS

$$L_{ac} = \frac{L}{\cos\theta} \qquad L_{bc} = L$$

WEIGHT OF TRUSS

γ = weight density of material

$$W = \gamma(A_{ac}L_{ac} + A_{bc}L_{bc})$$

$$= \frac{\gamma PL}{\sigma_{allow}}\left(\frac{1}{\sin\theta\cos\theta} + \frac{1}{\tan\theta}\right)$$

$$= \frac{\gamma PL}{\sigma_{allow}}\left(\frac{1+\cos^2\theta}{\sin\theta\cos\theta}\right) \qquad \text{Eq. (1)}$$

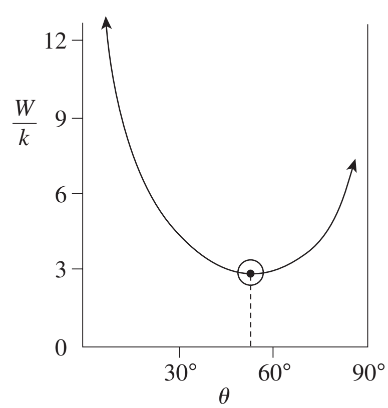
γ, P, L, and σ_{allow} are constants

W varies only with θ

Let $k = \dfrac{\gamma PL}{\sigma_{allow}}$ (k has units of force)

$$\frac{W}{k} = \frac{1+\cos^2\theta}{\sin\theta\cos\theta} \text{ (Nondimensional)} \qquad \text{Eq. (2)}$$

GRAPH OF EQ. (2):



Angle θ that makes W a minimum

Use Eq. (2)

Let $f = \dfrac{1 + \cos^2\theta}{\sin\theta\ \cos\theta}$

$$\frac{df}{d\theta} = 0$$

$$\frac{df}{d\theta} = \frac{(\sin\theta\cos\theta)(2)(\cos\theta)(-\sin\theta) - (1+\cos^2\theta)(-\sin^2\theta + \cos^2\theta)}{\sin^2\theta\cos^2\theta}$$

$$= \frac{-\sin^2\theta\cos^2\theta + \sin^2\theta - \cos^2\theta - \cos^4\theta}{\sin^2\theta\cos^2\theta}$$

Set the numerator $= 0$ and solve for θ:

$$-\sin^2\theta\cos^2\theta + \sin^2\theta - \cos^2\theta - \cos^4\theta = 0$$

Replace $\sin^2\theta$ by $1 - \cos^2\theta$:

$$-(1 - \cos^2\theta)(\cos^2\theta) + 1 - \cos^2\theta - \cos^2\theta \quad - \cos^4\theta = 0$$

Combine terms to simplify the equation:

$$1 - 3\cos^2\theta = 0 \qquad \cos\theta = \frac{1}{\sqrt{3}}$$

$\theta = 54.7° \longleftarrow$